Analysing and interpreting the yield curve

Moorad Choudhry

GBM Treasury

26 October 2011

Contents

Acknowledgements

Thanks to Nayan Sthanakiya (FICC SPT) for permission to use his slides on the recommended curve smoothing mechanism.

Thanks to Malgorzata Tynecka (GBMT Liquidity Management) for assistance with the charts.

Importance of the yield curve

Use of the yield curve

- Setting yield for capital market instruments
- Indicator of future economic conditions
- Measuring and comparing returns across term structure, and for different bonds of identical tenor (“relative value”)
- Pricing derivatives

Accurate and liquid input prices

- Value of curve reflects directly the accuracy and liquidity of the input prices
- Interpolation method is only as good as the input prices

Different interest rates

- Yield to maturity
- Zero-coupon rates
- Forward interest rate

No-arbitrage principle

Spot or zero-coupon rates

The true interest rate for any period *n* is the zero-coupon interest rate

(Note that "spot" rates are a theoretical construct but approach very short-term zero-coupon rates, in practice the terms are used interchangeably)

No-arbitrage principle

Interest rates available today must eliminate the possibility of arbitrage.

For example, the return from a 2-year bond must equal the return from buying a 1-year bond and rolling over (reinvesting) the proceeds for 1 year.

Simple illustration

The market term structure consists of the following only:

1-year rate	10%
2-year rate	12%

A customer wishes to fix the rate for borrowing 1-year money in 1-year's time. In a world of no bid-offer spreads and no additional credit spread, what rate does the bank quote? (Assume annual compounding)

[See the spot and forward rates "Formula Summary" at Appendix 1, which confirms that spot and forward rates are **saying the same thing**]

Discussion point

Two sides of the same coin

Assume a limited term structure environment of the form described on the last slide.

This represents your peer group COF curve – you do not fund materially cheaper or dearer than your peers.

You are happy to fund today in both 1 year and 2 year at these rates – the more the merrier.

A customer wishes to fix a 1-year 1-year forward starting deposit….

…

….Sorry, whats the question?

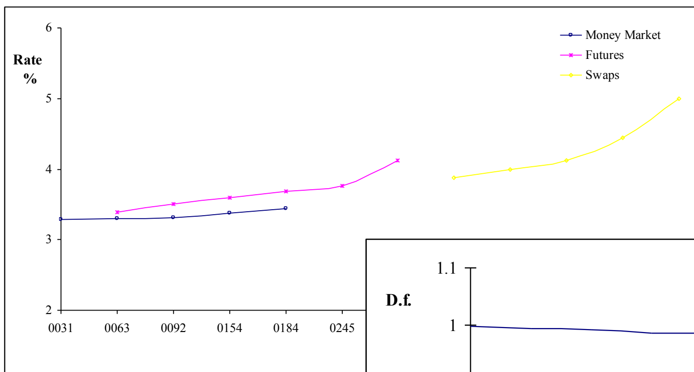
Constructing the curve from the zero-coupon discount function (simple linear interpolation)

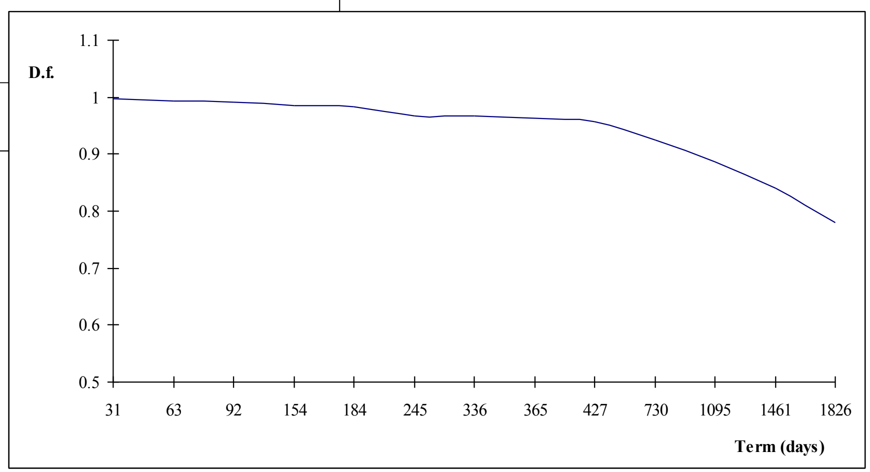
We take market rates comprising money market depos, futures prices and swaps. Example below is for GBP some time back…

Calculate the corresponding discount factor for each rate [Df= 1/(1 + (r x days/365)) or Df = $1/(1+r)^n$]

Use linear interpolation to calculate Df for those tenor points we don't have data for.

Problem arises for (i) moving from one data point class to another and (ii) when we connect the dots between far points





Days	Zero-coupon (%)	DF	Source
31	3.28	0.99718244	Money market
63	3.30	0.99426244	Money market
92	3.31	0.99160578	Money market
154	3.36	0.98514630	Futures
184	3.44	0.98273391	Money market
245	3.47	0.96792461	Futures
336	3.57	0.96772690	Futures
365	3.87	0.96269555	Swap
427	3.72	0.95775244	Futures
730	4.00	0.92451171	Swap
1095	4.13	0.88562084	Swap
1461	4.46	0.83969432	Swap
1826	5.09	0.78035608	Swap

Notes: calculate stub rate to first futures contract expiry by l.i. of 1m and 3mo MM rates

Assume swaps are par swaps, otherwise coupon payments at each period end need to be discounted back using Period 1 discount factor, to leave a zero-coupon structure

Whats wrong with this method? [2]

Theories of the yield curve

Expectations hypothesis

In essence this states that bondholders expectations determine the course of future interest rates

-- **Local expectations hypothesis** states that all bonds of the same class but differing in tenor will have identical expected *holding period rate of return*

-- ie., a 6-month and a 20-year bond will produce the same rate of return over a stated holding period

-- So of we wish to hold a bond for 6 months only, it doesn't matter which bond we hold as the return will be identical

-- Hence we prefer the **unbiased expectations hypothesis** which states that current implied forward rates are unbiased estimators of future spot interest rates. This isn't entirely accurate either but..

…as it states that the long-term interest rate is the geometric average of expected future short-term rates, so $(1+rs_n)^n = (1+rs_1)(1+{}_1rf_2)...(1+{}_{n-1}rf_n)$

-- it allows us to interpret a positively sloping curve as implying that spot interest rates will rise, and an inverted curve that spot interest rates will fall

So a rising yield curve is explained by investors expecting short-term interest rates to rise, that is

$$ {}_1rf_2 > rs_2 $$

(And the opposite for an inverted curve)

More on expectations…

Expectations are a function mainly of the expected rate of inflation

If the market expects inflationary pressures in the future, the curve will stay positively sloping

In theory then, in a benign inflationary environment one would observe something close to a flat yield curve, which reconciles with the return-to-maturity expectations hypothesis we have described already – return over time must equate to returns over short time rolled over

We don't observe either of the above very often!

As expected (pun intended), the expectations hypothesis doesn't explain every shape of the curve, so we need others…

But its logic does enable us to derive zero-coupon curves (spot curves) from coupon curves…

…and it articulates the no-arbitrage principle which is essential for finance pricing theory.

Liquidity preference theory

Liquidity preference theory explains the shape of the yield curve in terms of the conflict in investment horizons between borrowers and lenders

Speaking generally, borrowers prefer to fix their borrowing for as long as possible, while lenders wish to lend over as short a time as possible

This results in the market clearing to longer-dated rates being higher than shorter-dated ones, to compensate lenders for longer term risk

This implies a positive sloping curve at all times

An inverted curve could still be explained by liquidity preference when combined with the unbiased expectations hypothesis

The difference between a curve explained by unbiased expectations and an observed curve is the liquidity premium, the compensation required for holding longer-dated – and thereby less liquid – instruments

Described by

$$0 = L_1 < L_2 < L_3 < \ldots < L_n$$

$$\textit{and}$$

$$(L_2 - L_1) > (L_3 - L_2) > \ldots > (L_n - L_{n-1})$$

where *L* is the premium for a bond of *n* term, and states that the premium increases as the term to maturity increases, and that an otherwise flat curve will in fact be positively sloping.

Formal relationship

The spot and forward relationship in continuous time

In a continuous time environment we don't consider discrete time intervals over which interest is payable, but rather a period of time in which funds are borrowed and repaid instantaneously.

Spot rate is *r,(t, T)*. Just as before, the return generated by investing at forward rate *f (t, s)* over the period *(s – t)* must equal the return generated by investing initially at *r (t, T)* and rolling over. So we can set

$$e^{\int_t^T f(t,s)ds} = e^{r(t)dt}$$

which enables us to derive

$$r(t,T) = \frac{\int_t^T f(t,s)ds}{T-t}$$

This states that the spot rate is given by the ***arithmetic*** average of the forward rates *f(t, s)* where *t < s < T .*

Compare this to discrete time environment where the spot rate is the ***geometric*** average of the forward rates.

If we assume we are dealing today at time 0 for maturity at time *T*, the spot rate becomes

$$r(0,T) = \frac{\int_t^T f(0,s)ds}{T}$$

Formal relationship…

So we can write

$$r(0,T)T = \int_t^T f(0,s)ds$$

which is illustrated overleaf. It shows that the spot rate *r(0, T)* is the average of the forward rates from 0 to *T*.

We conclude then that as the spot rate increases, then by definition the forward rate (or *marginal* rate) will be greater. We can formalise a relationship stating that the forward rate is equal to the spot rate plus a value that is the product of the rate of increase of the spot rate and the time period. This is given by

$$f(t,s) = r(t,T) + (T-t)\frac{dr(t,T)}{dT}$$

This simply confirms the relationship we stated earlier in the discrete time environment.

The forward rate for any period will lie above the spot rate if the spot rate term structure is increasing, and will lie below the spot rate if it is decreasing.

In a constant spot rate environment, the forward rate will be equal to the spot rate.

Note the forward rate lies above it, but may itself be decreasing or increasing while the spot rate is increasing. As the spot rate is the average of the forward rates, it must accommodate this and so forward rates must be decreasing before the point at which the spot rate reaches its highest point.

Illustration of relationship between spot and forward rates

Diagrammatic representation of

$$r(0,T)T = \int_t^T f(0,s)ds$$

An example of decreasing forward rates as spot rates are still increasing.

All conventional and in order…but what value is there in knowing this?

Simply this: it helps us understand what spot and forward rates are telling us. If we see the logic it's built on, we can use it more appropriately

And it helps to interpret market expectations for economy

[BTW - the shaded area or the expression for it don't quite tally…one needs a slight correction!]

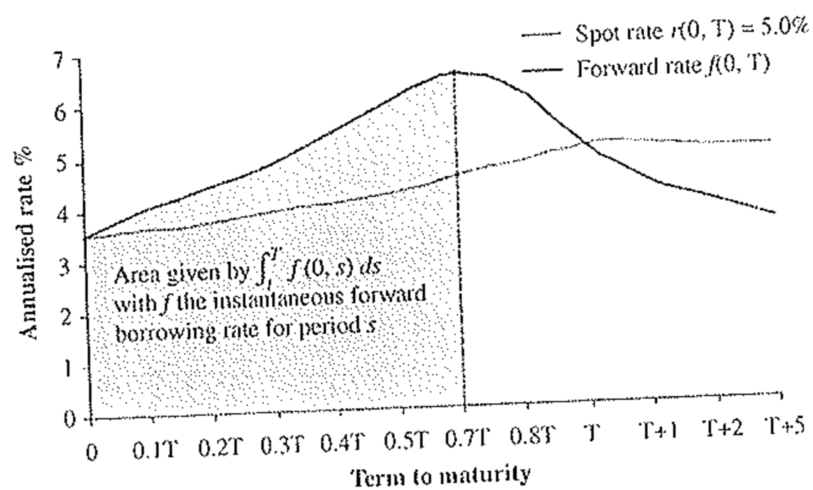


FIGURE 5.22 Diagrammatic representation of the relationship between spot and forward rate.

Apologies for the skew-iff exhibit…! [6]

Interpretation

Pointers to remember

The money market (0-12m or 0-24m) frequently acts independently of the capital market segment

Combining expectations hypothesis with the liquidity preference enables us to conclude:

-- The conventional shape in a stable economy is positively sloping. A steep slope can indicate a number of things…

-- Inverted curves signal expected fall in base rates, which suggests lower long-dated rates…in other words, an easing of monetary policy, which happens during a recession

-- Every US recession bar one since the war has been preceded by an inverted curve

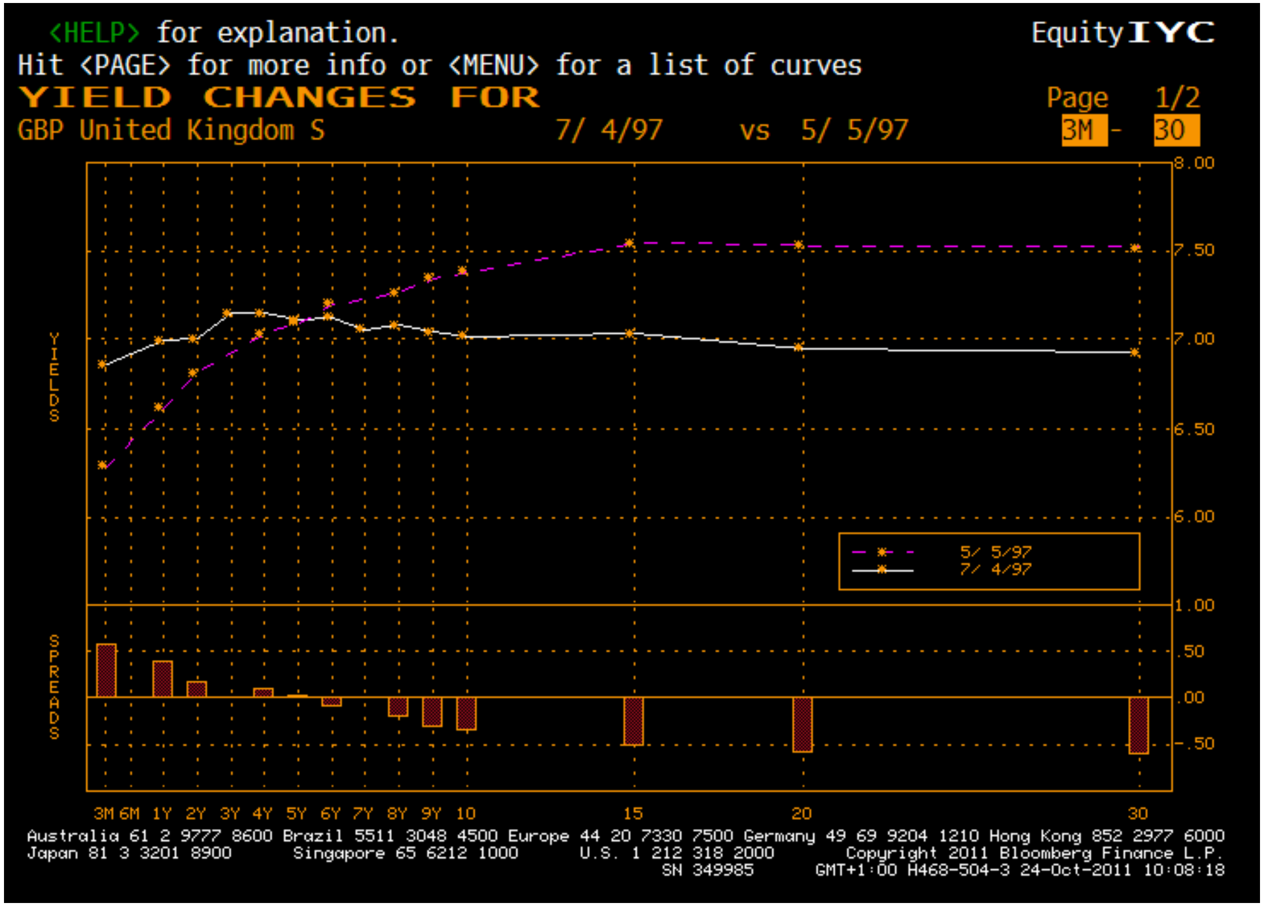
-- For a quick check, look up the curve for Jan 2007…!

-- Inflation expectations

-- Calculating the Liquidity Premium (compare curves of different issuer quality and amongst peers)

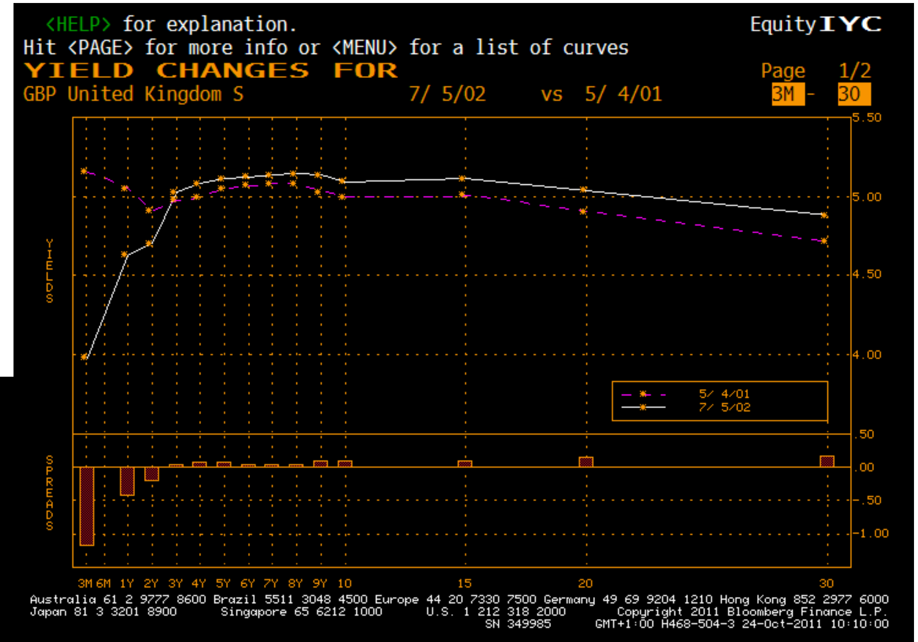
See also the examples overleaf, UK gilt curve – the curve can invert for technical reasons as well, or stay inverted for other non-macroeconomic reasons…

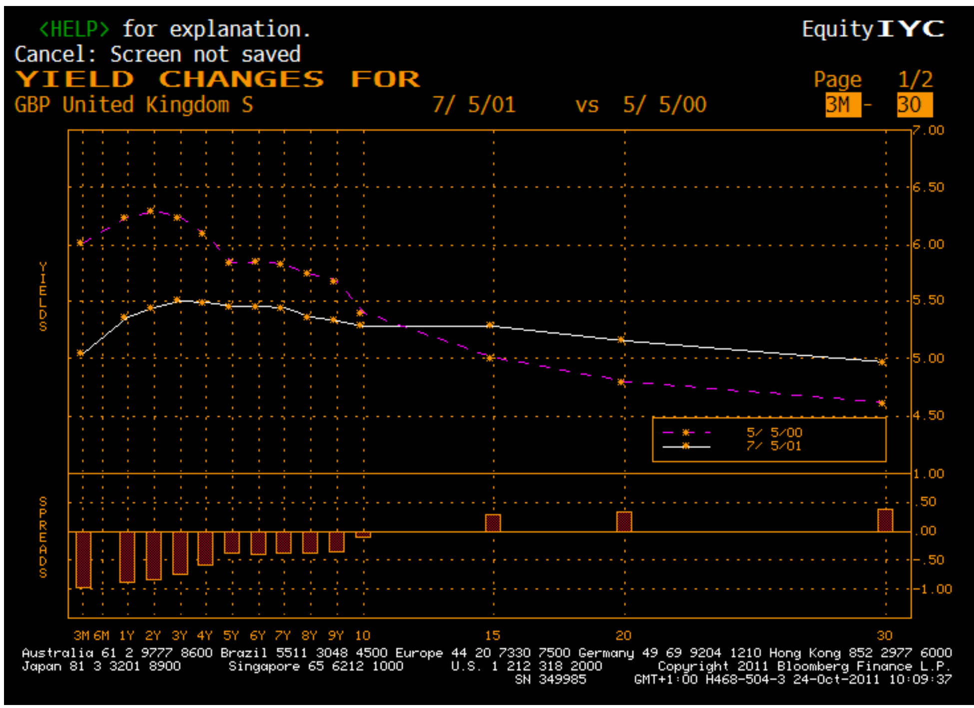
UK Gilt curves before and after 1997 election



UK gilt curves in positive economic conditions

- Still inverted in May 2000…
- …and May 2001…
- …finally positive in July 2001
- …and definitely so in July 2002



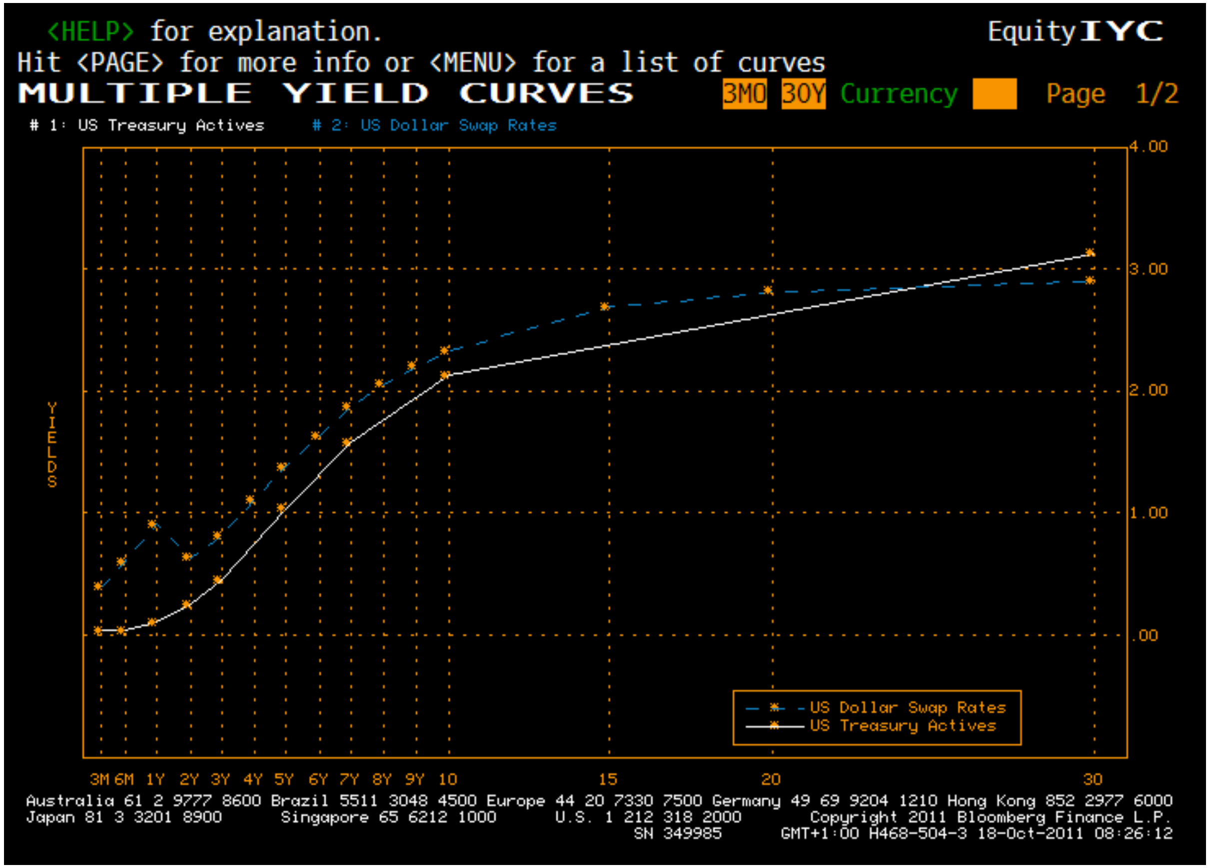


What about today?

US Treasury yield curve 18 Oct 2011

Why not inverted? Not expecting double-dip? What about the long-dated swap rate?

Answer to first may lie in the near-zero base rate…answers to the second?



Fitting the curve

The post-crash environment

The importance of coherent and robust internal funds pricing

- A sustainable business model will ensure that the cost of liquidity is determined correctly…
- …and passed on accurately.
- Pre-crash this was not in place at many institutions
- A move away from Libor-referencing to Cost of Funds (COF) referencing is one of the more obvious impacts of the liquidity crunch on bank internal procedures
- A COF curve, by definition, must be the baseline input to the FTP mechanism
- This then drives efficient, and correctly cost-based, pricing and risk management, resource allocation, business line transaction pricing, hedge construction and ROE/RAROC analysis

Background

- Orthodox valuation methodology in financial markets follows the logic of risk-neutral no-arbitrage pricing [1]. The same logic should therefore apply when setting a bank's internal pricing term structure.
- Banks now fund COF as opposed to Libor-flat; therefore, the logic of the no-arbitrage approach dictates that a bank's risky pricing yield curve should be extracted from market prices, because the latter dictate the rate at which the bank can raise liabilities.
- Such an approach preserves consistency because the same no-arbitrage principles drive market prices in the first place. In other words, the logic behind setting a bank's yield curve would be identical to the logic used when pricing derivatives.
- Business best-practice is to adopt an interpolation method that uses prices (yields) of the issuer's existing debt as model inputs, and extracts a discount function from these prices. The output is then used to derive a term structure that represents the issuer's current risky yield curve.
- To adopt such an approach requires a liquid secondary market in the issuer's bonds. This is a not available to every bank, however there are a number of proxy inputs and/or peer-group levels that can be used in such cases

Yield curve construction

Practical considerations

We can use the bootstrapping discount function approach – and extract the curve from this. If we had liquid secondary market bonds at regular 3 or 6 month intervals, out to 30 years, then linear interpolation would be just about acceptable (although still produce wildly oscillating forward curves)

As it happens, real world needs to consider:

-- lack of market data inputs, low liquidity levels

-- impact and volatility of bid-offer spreads

-- data impact of newer versus older bonds

Smoothing mechanisms

For these reasons practitioners need to apply an interpolation methodology more appropriate to the above

Interpolation methodology

The two most common interpolation methods in use are the cubic spline approach and the non-parametric approach. The former produces markedly oscillating forward rates and is also less accurate at the short-end [2], [3].

Therefore GBM has adopted the non-parametric method. The original non-parametric model is Nelson-Siegel [4], which is a forward rate model; however we recommend an extension of Nelson-Siegel for daily use, the Svensson (94) model, which produces a smoother forward curve, partly as a result of incorporating one extra parameter [5]. It also produces smoother short-date forwards.

[Model parameters given at Appendix 2]

Yield curve construction

GBM ALCO-approved procedure

- The recommended procedure is:

(1) Fit Svensson 94 to market data

(2) Extract the bank's risky yield curve. This is the baseline EUR yield curve that determines the fixed coupon for a vanilla coupon fixed-maturity bond issued at par

(3) Use the continuous discount function obtained from (2) above to create a par-par asset swap curve

(3) is the market-implied EUR Term Liquidity Premium (TLP) curve, which sets the spread for a vanilla FRN issued at par.

- The curve at (3) is therefore the bank's TLP as dictated by market rates. By definition, under the no-arbitrage principles we refer to above, this curve is the baseline pricing curve on the asset side as well, driving ROC.
- The baseline EUR curve is the source for creating all cross-currency funding curves.
- For smoothing purposes, calculate the 2-week (or 4-week) moving average TLP as well as the Spot TLP. The 2MA curve is the formal TLP, and preserves a more stable curve during periods of market volatility.

- [The curve construction procedure logic is described at Appendix 2]

An aside…

Defining “smooth”…

- There are pro’s and cons with each of the curve construction methodologies.
- There is also a mathematical definition of “smoothness”, such that one can derive, as opposed to argue, which method is the better fit (see Appendix 3)
- But before we address that, we should articulate EXACTLY what we want our curve to do:
  - Smoothest continuous forward rates
  - Smoothest discrete forward rates
  - Smoothest continuous zero-coupon yields
  - Smoothest discrete zero-coupon yields
  - Smoothest continuous zero-coupon bond prices
  - Smoothest discrete zero-coupon bond prices
  - Minimal pricing error
  - Combination of minimal pricing error and one of the other criteria
- …and then select the methodology that produces the smoothest result for the criteria we are after.
- So the method that produces the smoothest result mathematically is not necessarily the one to adopt at your particular bank

Operational issues

Long-dated input parameters

- The methodology requires liquid and observable secondary market bond yields to be completely efficient. The most accurate output will be that which matches the output tenor to the available input tenor.
- That said, many banks require long-dated TLP to set asset pricing and for ongoing FTP purposes.
- Where there is a lack of input data, the model output suffers from the lack of data points at the long end. This creates extreme volatility at the 30-year point.
- An issue arises with the stability of this long-dated output when there is no long- term issuance in existence. Slight changes in the inputs at the shorter end are magnified into significant changes at the long end. This is expected as it follows the logic of the interpolation methodology. However the paucity of input data results in output at the long end that is extremely volatile and therefore detrimental to the balance sheet valuation process.

- A proxy approach is to set a subjective 30-year bond yield as an input to the Svensson 94 procedure. This would be a robust market-implied input at the 30-year point, based on the bank's internal supply-and-demand level for such funds. This will ensure minimal oscillation of the forward rates and preserve the parametric methodology. In effect we are inserting an additional knot-point in the process, but at the final maturity point of the curve. This is akin to constraining the model output at its final point to the 30-year yield that is user-defined.

Proxy inputs

Alternative input data

- NOT CDS levels….post-crash this is not accurate..
- …positive basis pre-crash for banks has moved to negative basis post-crash. And back again..
- Peer bank levels
- Swap levels vanilla term against OIS term (a liquidity proxy to which we add a credit spread)
- A set of alternative curves, averaged

Dealing with market volatility

During periods of market volatility, daily changes to a bank's secondary market levels will create a volatile TLP

This may not be desirable for balance sheet management purposes

We recommend a switch in the 2-week MA to a 6-week MA during such periods

A number of indicators can be used to determine when the market is now in "volatile" or "stressed" condition

One such indicator is the EONIA versus 3-mo Euribor spread

Switching regime methodology

We set two metrics, based on the bank's asset swap spread and the selected market indicator (EONIA-Euribor)

We have:

- Alpha: the 1-week standard deviation for 5-year ASW, normed relative to (say) 15 and capped at 1.00
- Beta: the EONIA-Euribor basis swap, normed relative to a set level (say current level of 50 bps), capped at 1 and floored at 0
- Mixing parameter: M = Alpha*Beta
- The output mid is the convex combination of the 2-week MA and the 6-week MA of Svensson spot

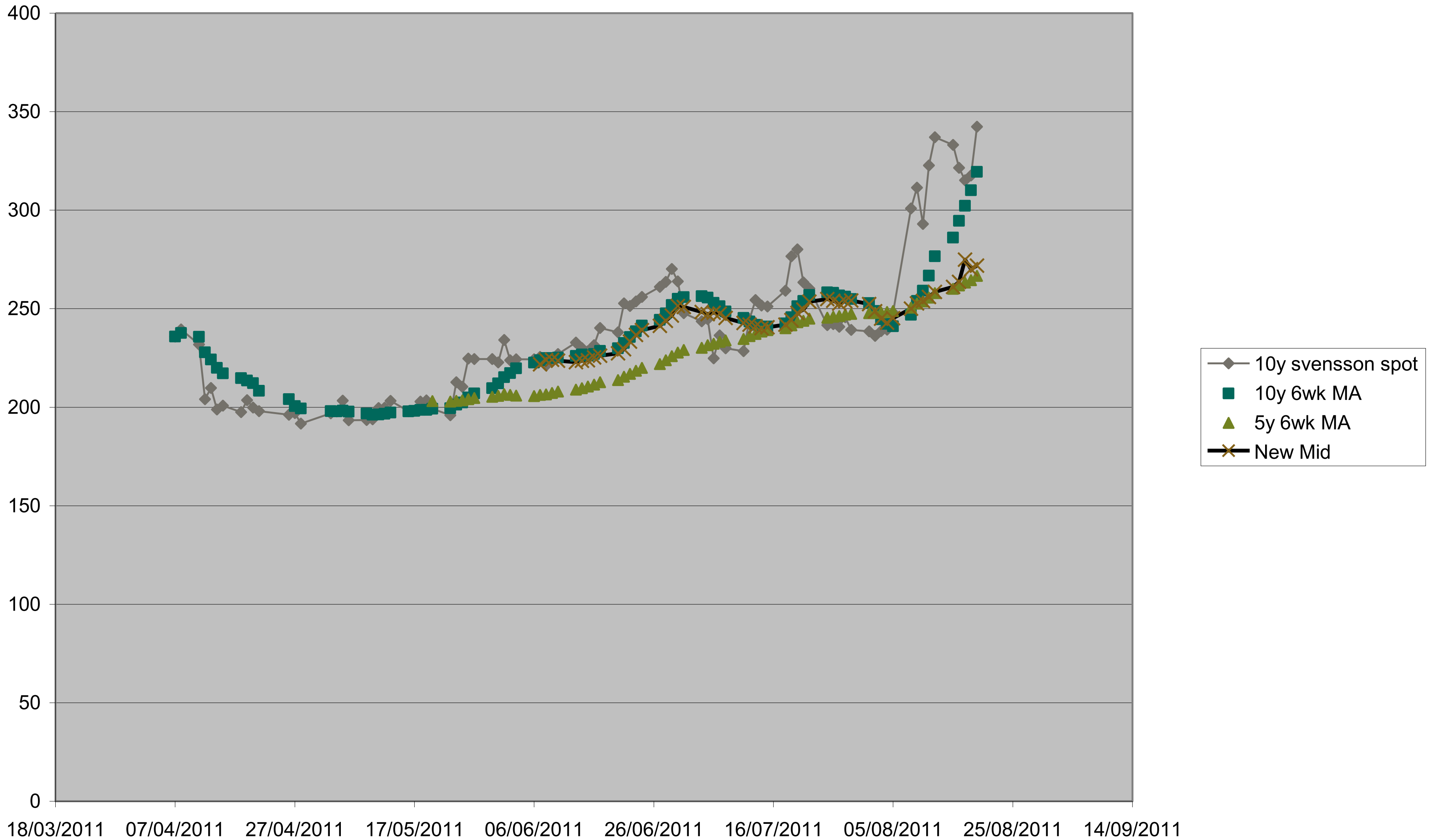
The new curve output is given by:

- M*6MA + (1-M)*2MA

See the charts overleaf for the smoothing impact

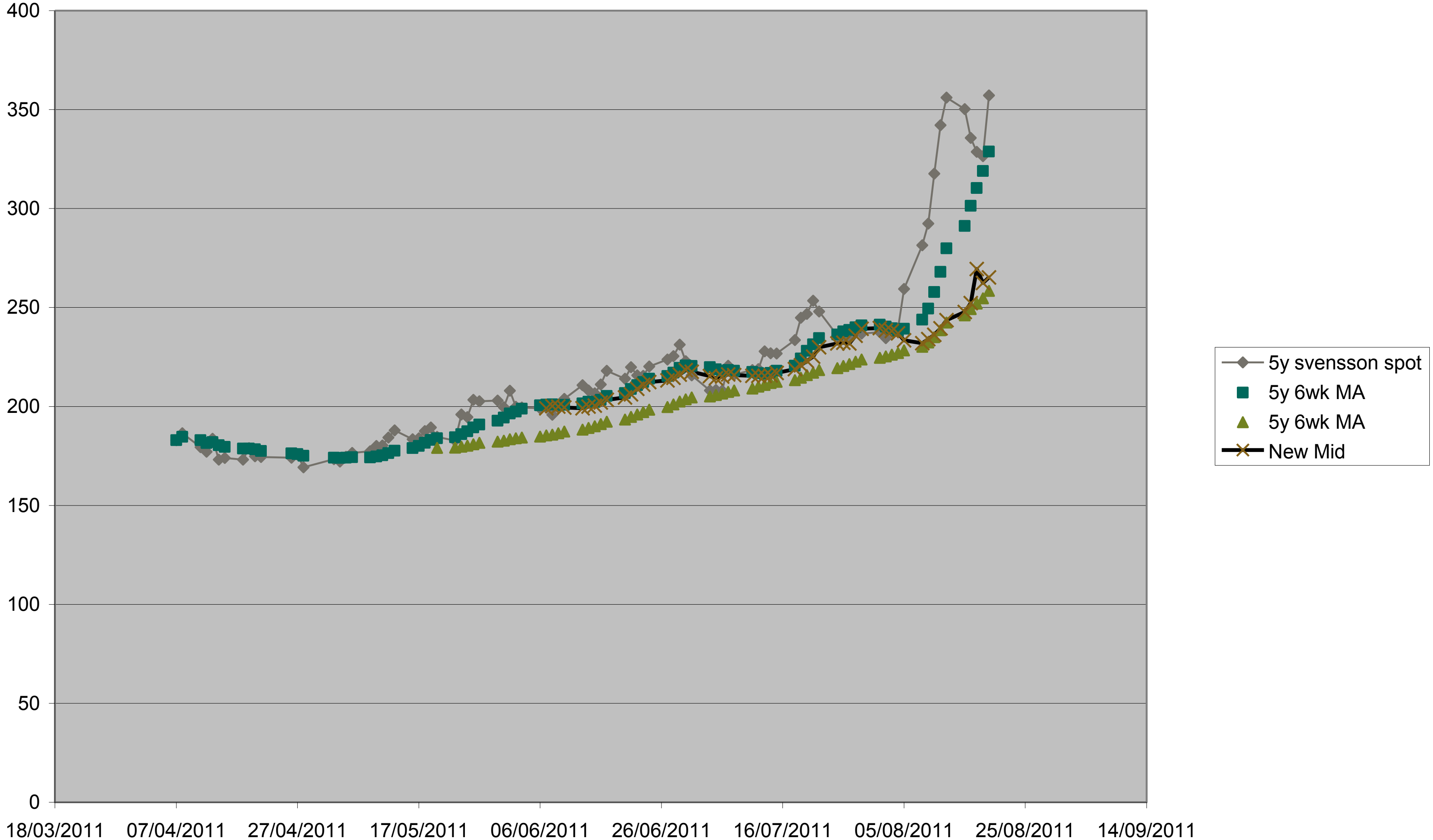
The alternative is simply to select a 14-day or 60-day MA curve every day, and stick with that. User can move to the longer MA period during periods of market volatility.

2-year point



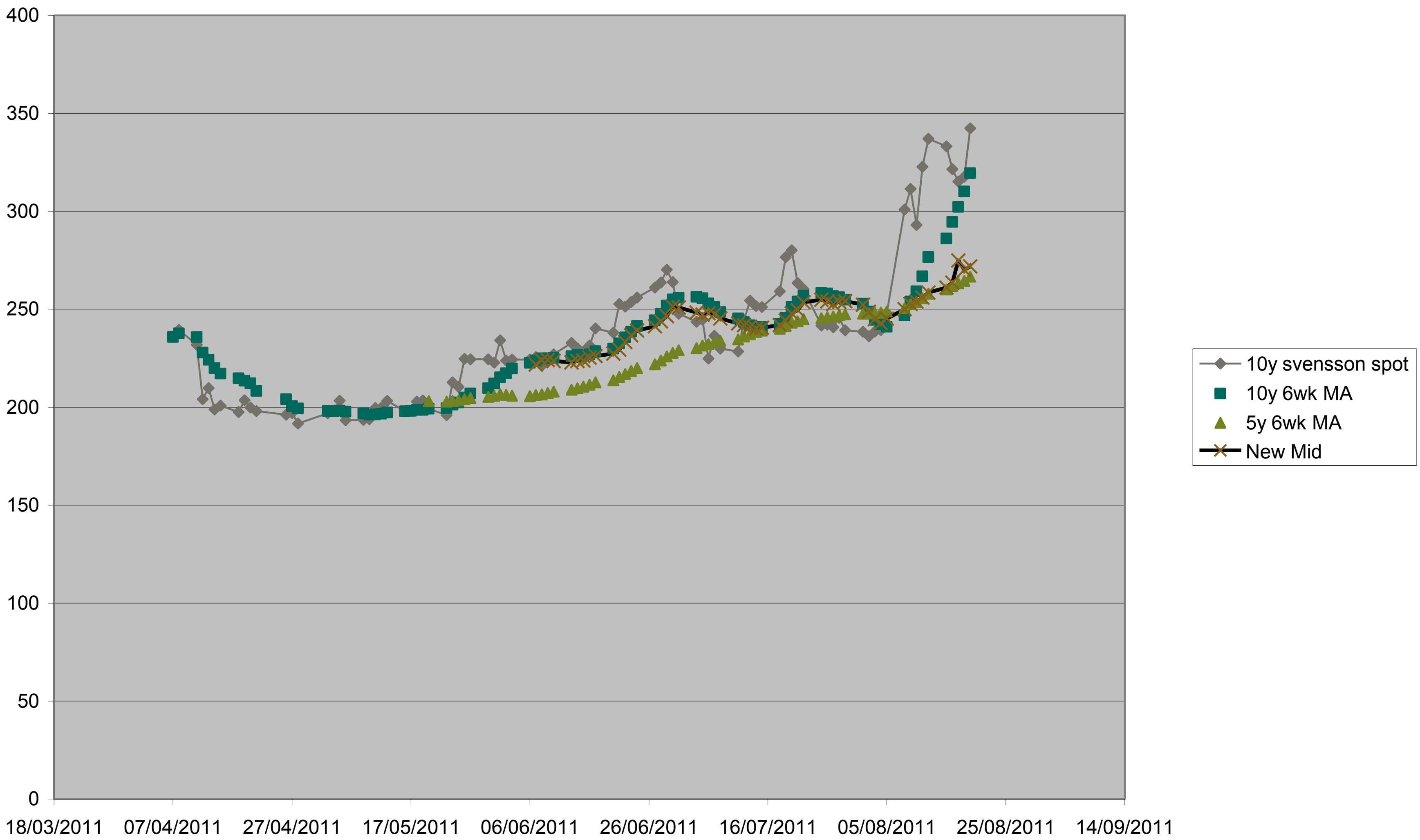
RBS

5-year point



RBS

10-year point



Appendix 1

Formula Summary

The spot yield is the is the geometric mean of the forward rates, given by

$$\left(1 + rs_n\right) = \left(1 + {}_0rf_1\right)\left(1 + {}_1rf_2\right)\ldots\left(1 + {}_{n-1}rf_n\right)$$

which implies the following relationship between spot and forward rates

$$\left(1 + {}_{n-1}rf_n\right) = \frac{\left(1 + rs_n\right)^n}{\left(1 + rs_{n-1}\right)^{n-1}}$$

$$= \frac{df_{n-1}}{df_n}$$

Appendix 2

Svensson 94 model

- In the Svensson model, the instantaneous forward rate of n maturity at time t is given as

$$f_t(n,0) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2(n/\tau_1)\exp(-n/\tau_1) + \beta_3(n/\tau_2)\exp(-n/\tau_2)$$

b_0	*Long-run levels of interest rates*
b_1	*Short-run component*
b_2	*Medium-term component 1*
b_3	*Medium-term component 2*
t_1	*Decay parameter 1*
t_2	*Decay parameter 2*

- The parameters are user-specified or otherwise left for the model to set to best fit.
- The output extrapolates from input where no equivalent input data point is entered

Appendix 2

Detailed curve construction procedure and logic

- We desire to extract the credit-risky curve from market prices. To do this we require a liquid secondary market of the bank's bonds, and an interpolation model. Practitioners generally use either the cubic spline approach or a non-parametric model approach.
- The procedure we recommend involves the following:

1 – Extract the risky yield curve using the bank's money-market funding rates and prices of secondary market bonds. We set the model's Beta and Tau parameters ourselves, or otherwise allow for the model to extract the ordinary least-squares best fit. The parameters include the long-run expected interest rate, which in general would be user-specified. This will be set as part of regular discussion within Treasury and ALCO.

2 – Running the model produces an interpolated curve based on market inputs (see Exhibit 1), and a discount function in near-continuous time (see Exhibit 2, which shows the function in annual time steps. This can be adjusted for monthly or daily time steps if desired). This is the Svensson discount function (DF). We convert this DF to match to EUR swap dates, and to spot settlement.

3 – We extract the par yield curve. We now have a set of discrete rates corresponding to the bank's fair value yield curve, which tells us the coupon to set on a vanilla fixed coupon bond we issue at par for the relevant tenor.

4 – We use these rates to construct a full yield curve.

Continued….

Appendix 2….

- From the curve at (4) we extract the implied TLP. (Exhibit 3) This shows the fair-value spread the bank would in theory pay on an FRN it issued of relevant tenor with a floating re-set of 3 months Euribor.

- (Note: this also sets the rate to pay on a fixed coupon bond issue that was asset-swapped, but for an unsecured swap [as this is the unsecured pricing curve]. Hence it would not be the correct fair-value spread to pay on an asset swap, because that would involve a secured derivative. If we assume an unsecured derivative, we now have the par-par asset swap curve.).

- We should note that the above is the market-implied curve process. The model output extrapolates beyond the latest tenor of existing issued bonds to as long a maturity as the user wishes, and as a function of the long-run expected forward rate. Exhibit 3 shows yields up to 10-year. The user can set the model to extrapolate fair-value output to a tenor of its choice.

- Note that extrapolated rates represent the current secondary market-implied value, and hence the fair-value rate to pay for that tenor. The actual rate paid at any one time will also reflect business considerations and the impact of supply and demand. That is, a specific business case can be made at any time as to why specific tenor points at any point along the TLP curve should deviate by more than the accepted tolerance stated in this procedure.

Appendix 2 Exhibits

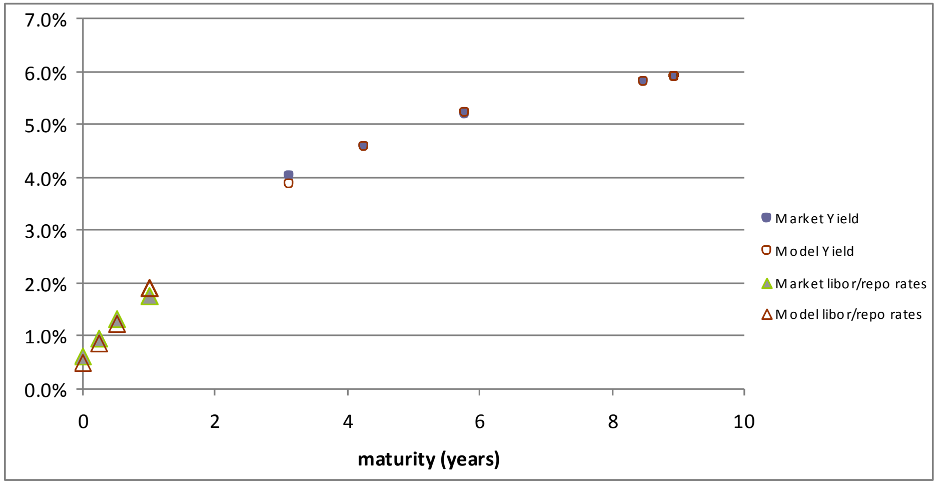


Exhibit 1: Market/Model yield-to-maturity

Maturity (year)	*discount factor*	*forward rates*	*spot rates*
0	1	0.48%	0.48%
1	0.981433988	3.10%	1.87%
2	0.942635953	4.85%	2.95%
3	0.892551064	5.98%	3.79%
4	0.837510049	6.68%	4.43%
5	0.781645775	7.08%	4.93%
6	0.727458537	7.26%	5.30%
7	0.676326573	7.30%	5.59%
8	0.628899097	7.23%	5.80%
9	0.585375249	7.10%	5.95%
10	0.545690715	6.93%	6.06%

Exhibit 2: Svensson 94 discount function, annual time steps

Point	**implied 3m spreads**
3m	**-17**
6m	**-1**
1y	**33**
18m	**66**
2y	**95**
3y	**147**
4y	**184**
5y	**210**
6y	**228**
7y	**240**
8y	**248**
9y	**253**
10y	**255**
Exhibit 3 Extracted implied TLP	**254**

Appendix 3

Defining smoothness

- Smoothness refers to the extent to which a mathematical function oscillates.
- A smoothess penalty is defined as

$$\int \left(f''(t)\right)^2 dt$$

- Or as absolute values

$$\int \left|f'(t)\right| dt$$

- Zero-coupon rates are averages of forward rates….smoothing forward rates by definition produces smoother zero rates. The criteria for smoothness in forward rates is given by

$$f_1 = f(t_0, t_1), f_2 = f(t_1, t_2), \ldots, f_n = f(t_{n-1}, t_n)$$

- Moving to a discrete measure in which forward rates are as near to their previous rate as possible, so that the following should be minimised

$$\sum_{i=2}^{n} \left|f_i - f_{i-1}\right|$$

- In practice we use the squared differences for calculations

$$S(f) = \sum_{i=2}^{n} \left(f_i - f_{i-1}\right)^2$$

References

- [1] The most appropriate references in this field are Feynman-Kac (1949), Ito (1951), Markowitz (1959), Fama (1970), Black-Scholes (1973) and Merton (1973).
- [2] James, J., and N. Webber (2000), *Interest Rate Modelling*, Chichester: John Wiley & Co Ltd
- [3] Choudhry, M. (2003), *Analysing and Interpreting the Yield Curve*, Singapore: John Wiley & Sons Pte (Asia) Ltd
- [4] Nelson, C., and A.F. Siegel, (1987), "Parsimonious Modeling of Yield Curves", *Journal of Business*, 60, pp.473-489
- [5] Svensson, Lars E. O. (1994), "Estimating and Interpreting Forward Rates: Sweden 1992-4," *National Bureau of Economic Research Working Paper #4871*
- [6] Choudhry, M., (forthcoming), *The Principles of Banking*, Singapore: John Wiley & Sons

This presentation is for internal use only.

But if you use it externally, the following Disclaimer applies…