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Accessible approach to estimation of the Nelson-Siegel Yield Curve

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Term Structure Modelling Using the Nelson-Siegel Model

The Nelson-Siegel model offers an accessible and tractable framework to model the interest rate curve. In essence we wish to fit the empirical form of the yield curve with a pre-specified functional form, of the following:

$$y(m) = \beta_o + \beta_1 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau}\right) + \beta_2 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau}\right)$$

The Nelson-Siegel Model following some adjustments provided by Prof. Moorad Choudhry is:

$$y(m) = \beta_o - \beta_1 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau}\right) + \beta_2 \cdot \left(\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau}\right)$$

where:
y is the spot rate and *(m)* is the maturity

β_0, β_1, β_2 and τ are parameters to be fitted via a least-squares algorithm. These parameters can then be translated into shift, twist and butterfly movements.

The main objective of the spreadsheet implementation is to estimate the coefficients of the Nelson & Siegel model using Microsoft Excel and VBA. An example of the output is illustrated later in this account.

Nelson-Siegel Estimation (Maturity, Yield)

Maturity is a vector of maturity dates.

Yield is a vector of interest rates for the maturity dates of the former parameter.

Remarks: This function returns the four parameters β_0, β_1, β_2 and λ_t.

Nelson-Siegel Yield (Maturity, Level, Slope, Curvature, Tau) [1]

Yield based on the Extended Nelson & Siegel Model

Maturity is the maturity of the interest rate that must be estimated.

Level is the parameter β_0 in the above equation and interpreted as the long run levels of interest rates.

Slope is the parameter β1 in the above equation and the short-term component.

Curvature is the parameter β_2 in the above equation and the medium-term component.

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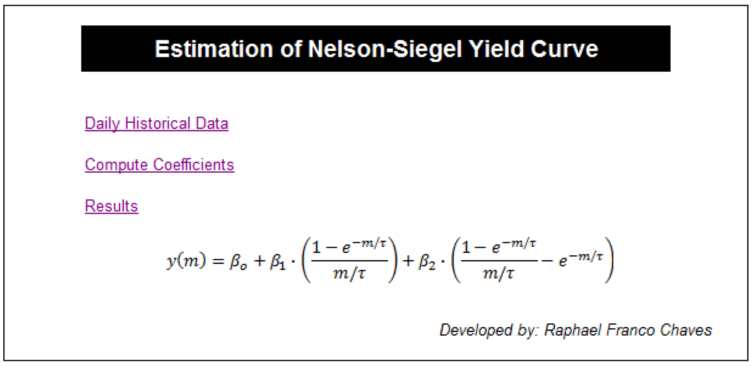
[1] For a theoretical presentation of this approach see:
Charles R. Nelson, Andrew F. Siegel
Parsimonious Modeling of Yield Curves
The Journal of Business, Volume 60, Issue 4 (Oct., 1987), 473-489
Document available in: http://www.math.ku.dk/~rolf/teaching/NelsonSiegel.pdf

Tau is the parameter τ in the above equation and represents the decay factor: small values produce slow decay and can better fit the curve at long maturities, while large values produce fast decay and can better fit the curve at short maturities

For the first step one updates the Daily History of Interest Rates. After selecting the Daily Historical Data sheet, the user will be directed for the database of the spreadsheet.



The user can select any Yield to Maturity. There is no tenor for time to maturity. For example, the following illustrates:

Back	SF0001W	SF0001M	SF0003M	SF0006M	SF0012M	SFSW2	SFSW3	SFSW4	SFSW5	SFSW7	SFSW10	SFSW15	SFSW20	SFSW30
Time to Maturity ->	0,0192	0,0833	0,2500	0,5000	1,0000	2,0000	3,0000	4,0000	5,0000	7,0000	10,0000	15,0000	20,0000	30,0000
2001-01-01	3,3683	3,3667	3,3700	3,3700	3,3683	3,3850	3,4425	3,4950	3,5550	3,7050	3,9150	4,1100	4,2000	4,2800
2001-01-02	3,3533	3,3483	3,3483	3,3483	3,3483	3,2500	3,3200	3,3675	3,4325	3,5925	3,8225	4,0225	4,1075	4,1825
2001-01-03	3,4283	3,3550	3,3233	3,2983	3,2783	3,2350	3,2700	3,3300	3,3950	3,5500	3,7700	3,9650	4,0550	4,1400
2001-01-04	3,4767	3,3350	3,3050	3,2400	3,2250	3,2100	3,2450	3,2950	3,3600	3,5175	3,7525	3,9525	4,0375	4,1225
2001-01-05	3,5000	3,3917	3,3533	3,2917	3,2350	3,1900	3,2200	3,2750	3,3350	3,4825	3,7025	3,8975	3,9875	4,0725
2001-01-08	3,5000	3,3650	3,3217	3,2400	3,1717	3,1950	3,2275	3,2750	3,3325	3,4775	3,6925	3,9025	3,9975	4,0775
2001-01-09	3,6000	3,4433	3,3867	3,2983	3,2467	3,2500	3,2800	3,3300	3,3975	3,5475	3,7675	3,9650	4,0675	4,1400

For the next step one computes all coefficients in the model. The user then selects “Compute all Coefficients” button and that then produces the results, an example of which is shown below:

Term Structure Modelling Using the Nelson-Siegel Model

Back

Estimation of Nelson-Siegel Yield Curve

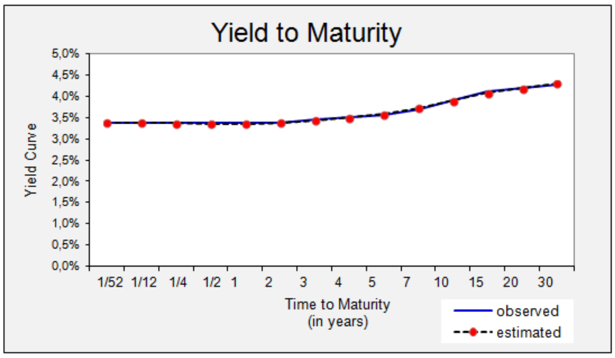
level	4,52%
slope	1,14%
curvature	-1,51%
tau	2,51

RMSE
0,0000%

4
2001-01-01

Compute all Coefficients

Number of Terms	Time to Maturity	Yield Curve		
YTM	(in years)	observed	estimated	Error
1	1/52	3,37%	3,38%	0,0000%
2	1/12	3,37%	3,38%	0,0000%
3	1/4	3,37%	3,37%	0,0000%
4	1/2	3,37%	3,36%	0,0000%
5	1	3,37%	3,35%	0,0000%
6	2	3,39%	3,38%	0,0000%
7	3	3,44%	3,43%	0,0000%
8	4	3,50%	3,50%	0,0000%
9	5	3,56%	3,58%	0,0000%
10	7	3,71%	3,72%	0,0000%
11	10	3,92%	3,90%	0,0000%
12	15	4,11%	4,08%	0,0000%
13	20	4,20%	4,19%	0,0000%
14	30	4,28%	4,30%	0,0000%



The Parameters Level, Slope, Curvature and Tau are available in the Results Spreadsheet. The Graphs are automatically updated in accordance with the information provided in the Daily Historical Data Spreadsheet.